

From examples to generalization: learning mathematics through problem solving

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Abstract: This presentation presents findings from the author's previous research in the field of mathematical problem solving, inductive reasoning, and generalization in mathematics education. The studies explore how learners of different ages and educational backgrounds develop mathematical reasoning while solving open-ended mathematical problems, and how teachers can effectively support these processes.

The first study compares the competences in inductive reasoning of primary teacher students and mathematics teacher students when solving a geometrical problem involving spiral patterns. The findings reveal differences in the depth of generalization, problem-solving strategies, and the ability to recognize mathematical structures. *The second study* examines pupils' problem-solving processes in a guided learning environment. Primary and secondary school pupils solve geometrical problems while being observed and supported by prospective primary teachers. The results show that most pupils need carefully designed conceptual guidance in order to progress from concrete examples toward more general mathematical reasoning, while older pupils are more successful in reaching algebraic generalizations. *The third study* investigates the relationship between types of generalization and the development of problem-solving schemas among primary teacher students. The findings demonstrate that successful generalization is closely connected to well-structured cognitive schemas and to the ability to connect mathematical concepts with contextual problem situations.

Together, the presented studies emphasize the importance of problem solving as a central component of mathematics education and teacher preparation, particularly for fostering reasoning, generalization, and deeper mathematical understanding.